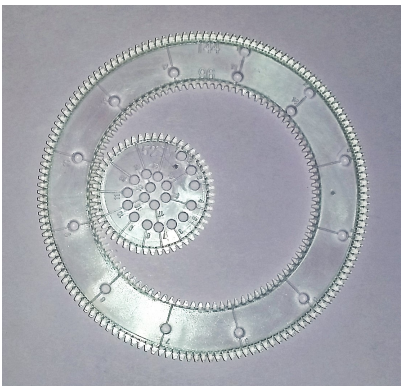


# Math 10460 - Honors Mathematics II

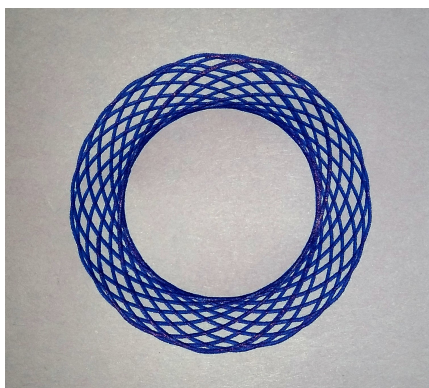
## Project 1 - Spirographs

### Spring 2016

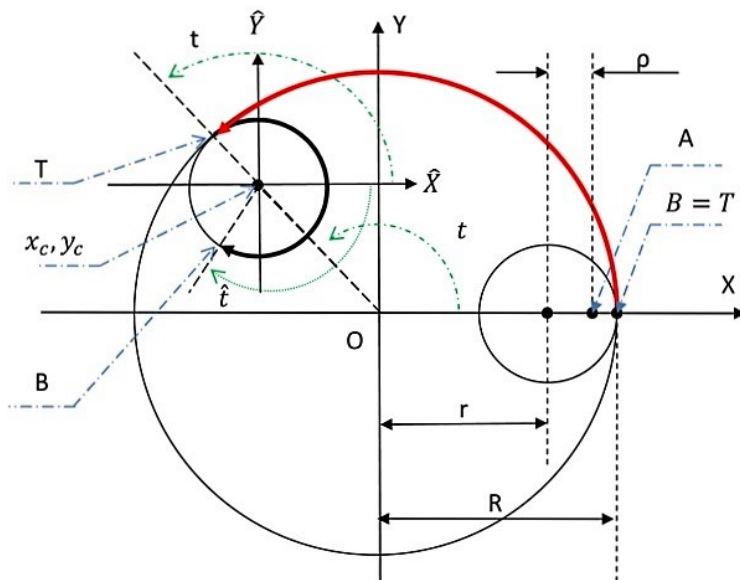
This project involves the popular toy called “Spirograph.” The toy consists of a ring and a small disk with holes in it for a pen. You use your pen to roll the disk around the inside (or outside) of the ring to create a pattern. Here is a picture of a Spirograph:



and an example of a picture you can create with it:



Here is a website on which you can draw Spirograph pictures: <http://nathanfriend.io/inspirograph/>. The idea behind a Spirograph is that you have one big circle and a smaller circle rolling around the inside edge of the big circle. Choose a point inside the smaller circle which is not on the edge and is not the center and trace its trajectory as the smaller circle rolls around. This is how a Spirograph drawing is created.



(From <http://theartinscience.blogspot.com/2010/07/whats-deal-with-spirographs.html>)

While making these drawings are enough fun on their own, we can enhance the fun by learning how they work mathematically!! In this project you will learn how this toy works and you will produce a computer model of a simple spirograph. There are two parts to this project:

- (1) Give the derivation of a mathematical description of a Spirograph picture. You can give it in any way you deem appropriate: a polar equation of the form  $r = f(\theta)$ , a set of parametric equations  $x = x(t), y = y(t)$  (which I think will be easiest), a vector equation  $\vec{r}(t) = \langle x(t), y(t) \rangle$ , or anything else you see fitting.
- (2) Write a Mathematica program which will sketch Spirograph curves for various sizes of the outer and inner circles, and for various locations of the point inside the smaller circle.

## MATHEMATICA COMMANDS

Mathematica is, by and large, pretty intuitive, I have found. There are many other commands, but this should cover the essentials for this project.

To graph parametric curves, use the ParametricPlot command:

$$\text{ParametricPlot}[\{x(t), y(t)\}, \{t, a, b\}]$$

This will sketch the curve with parametric equations given by  $x = x(t), y = y(t)$  for  $a \leq t \leq b$  (See <http://reference.wolfram.com/language/ref/ParametricPlot.html>) To create sliders, use the Manipulate command (to get  $\pi$  in Mathematica hit ESC, type “pi”, then hit ESC one more time):

$$\text{Manipulate}[\text{ParametricPlot}[\{t, \text{Sin}[n * t]\}, t, 0, 2\pi], \{\{n, 1\}, 0.1, 10\}]$$

This will plot  $y = \sin nx$  over the interval  $[0, 2\pi]$  (as a parametric curve!), and will give you a slider for  $n$  to control the period. It has an initial value of  $n = 1$  (the number inside the inside brackets, next to  $n$ ), and  $n$  can range from 0.1 to 10. You can set up many variables to be manipulated inside of one Manipulate command, e.g.

$$\text{Manipulate}[\text{ParametricPlot}[\{t, \text{Sin}[n * t]\}, t, 0, s], \{\{n, 1\}, 0.1, 10\}, \{\{s, 2\pi\}, \pi, 4\pi\}]$$

will let you control the period (with  $n$ ), and the length of the interval the curve is plotted over (with  $s$ ). (See <http://reference.wolfram.com/language/ref/Manipulate.html>)